

Stability of Bose Einstein condensates of hot magnons in YIG

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(Dated: February 2, 2008)

We investigate the stability of the recently discovered room temperature Bose-Einstein condensate (BEC) of magnons in Yttrium Iron Garnet (YIG) films. We show that magnon-magnon interactions depend strongly on the external field orientation, and that the BEC in current experiments is actually metastable - it only survives because of finite size effects, and because the BEC density is very low. On the other hand a strong field applied perpendicular to the sample plane leads to a repulsive magnon-magnon interaction; we predict that a high-density magnon BEC can then be formed in this perpendicular field geometry.

PACS numbers:

In a remarkable very recent discovery [1], a *Room Temperature* Bose-Einstein condensate (BEC) of magnon excitations was stabilized for a period of roughly 1 μ s in a thin slab of the well-known insulating magnet Yttrium Iron Garnet (YIG). The density of magnons was quite low ($n \sim 10^{-4}$ per lattice site), and the density n_o of the BEC was apparently unknown, but $n_o/n \ll 1$. This result can be understood naively in terms of a weakly-interacting dilute gas of bosons, provided that (i) one assumes that the number of magnons is conserved, so their chemical potential may be non-zero, and (ii) the interactions between them are repulsive (attractive interactions favour depletion of the BEC, causing a negative compressibility and instability of the BEC [2]). In the experiment, the magnon dispersion was controlled both by the sample geometry and external magnetic field, in such a way that magnon-magnon collisions conserved magnon number - the decay of the BEC was then attributed to spin-phonon couplings [1]. The magnons in the experiment had rather long wavelengths, of order a μ m - hitherto such excitations have been treated entirely classically [3].

This experiment raises a number of important questions, not least of which concern the kind of superfluid properties possessed by a BEC of such unusual objects. However a much more basic question about the stability of the system must first be answered. In fact we find the rather startling result that under the conditions of the experiments reported so far, these interactions were actually *attractive*, ie., the BEC ought to be unstable! We shall see that because of the particular geometry used, the BEC is actually metastable to thermally activated or tunneling decay, and that it only survives because its density is low - above a critical density, given below, it is absolutely unstable. However we also find that by changing the field configuration in the system one can make the interactions repulsive, and the BEC should then stabilize at a much higher density - opening the way to much more interesting experiments.

YIG is one of the best characterized of all insulat-

ing magnets [4]. It is cubic, with lattice constant $a_o = 12.376 \text{ \AA}$, ordering ferrimagnetically below $T_c = 560 \text{ K}$. At room temperature the long-wavelength properties can be understood using a Hamiltonian with ferromagnetic exchange interactions between effective 'block spins' \mathbf{S}_j , one per unit cell, whose magnitude $S_j = |\mathbf{S}_j| = a_o^3 M_s / \gamma$, with $\gamma = g_e \mu_B$, is defined by the experimental saturated magnetisation density M_s ($M_s \approx 140 \text{ G}$ at room temperature; with $g_e \approx 2$, one has $S_j \approx 14.3$), along with dipolar couplings between these; the resulting lattice Hamiltonian takes the form:

$$\begin{aligned} \hat{H} = & -\gamma \sum_i \mathbf{S}_i \cdot \mathbf{H}_o - J_o \sum_{i,\delta} \mathbf{S}_i \mathbf{S}_{i+\delta} \\ & + U_d \sum_{i \neq j} \frac{\mathbf{S}_i \mathbf{S}_j - 3(\mathbf{S}_i \cdot \mathbf{n}_{ij})(\mathbf{S}_j \cdot \mathbf{n}_{ij})}{|r_{ij}|^3}, \end{aligned} \quad (1)$$

where the sums i, j are taken over lattice sites at positions \mathbf{R}_i , etc., δ denotes nearest-neighbor spins, $\mathbf{r}_{ij} = (\mathbf{R}_i - \mathbf{R}_j)/a_o$, and $\mathbf{n}_{ij} = \mathbf{r}_{ij}/|\mathbf{r}_{ij}|$. The nearest-neighbour dipolar interaction $U_d = \gamma^2/a_o^3 \approx 1.3 \times 10^{-3} \text{ K}$. The isotropic exchange J_o is determined experimentally from $J = 2S J_o a_o^2 \approx 0.83 \times 10^{-28} \text{ erg cm}^2$ at room temperature. One then has $J_o \approx 1.37 \text{ K}$, and $U_d/J_o \approx 0.95 \times 10^{-3}$.

In what follows we set up a theoretical description of the BEC, taking into account the external field, dipolar and exchange interactions, and boundary conditions in the finite geometry. We evaluate the interactions and the BEC stability for 2 different field configurations; the general picture then becomes clear.

(i) In-plane field: All experiments so far have had \mathbf{H}_o in the slab plane. The combination of exchange, dipolar, and Zeeman couplings then leads to a magnon spectrum $\omega_{\mathbf{q}}$ shown in Fig.1, in which the competition between dipolar and exchange interactions leads to a finite- q minimum in $\omega_{\mathbf{q}}$ at a wave-vector $Q \sim 1/d$, where d is the slab thickness. To completely specify $\omega_{\mathbf{q}}$ and the inter-magnon interactions one needs boundary con-

ditions, which can involve partial pinning of the surface spins [5, 6, 7]. Demokritov et al [1] assume free surface spins, implying that (i) $|(\partial M(\mathbf{r})/\partial \mathbf{r}) \cdot \mathbf{n}_s| = 0$ when \mathbf{r} is at the surface; here \mathbf{n}_s is the normal to the surface, and (ii) that the allowed momenta along \hat{z} (see Fig.1, inset) are $q_\perp = n_\perp \pi/d$, leading to different magnon branches labelled by n_\perp . For now we assume a continuous in-plane momentum, and later discuss the effect of in-plane quantization; and we assume $n_\perp = 0$, taking the lowest-energy magnon branch.

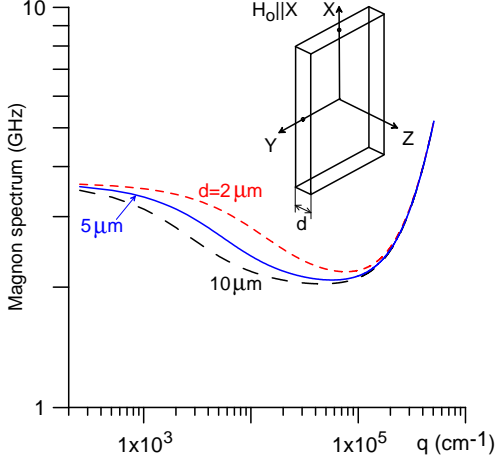


FIG. 1: (color online). The magnon spectrum, Eq.2, for $d = 2, 5$ and $10 \mu m$ at $H_o = 700$ Gauss. The inset shows the sample geometry.

The above assumptions yield an $\omega_{\mathbf{q}}$ which agrees with experiment [1], and which for $\mathbf{H}_o \parallel \hat{x}$ takes the form [7]

$$\hbar\omega_{\mathbf{q}} = [(\gamma H_i + Jq^2)(\gamma H_i + Jq^2 + \hbar\omega_M \mathcal{F}_q)]^{1/2}, \quad (2)$$

here $H_i = H_o - 4\pi N_x M_s$ is the internal field, N_x the demagnetization factor (for a slab in the xy -plane, $N_x = N_y = 0$, $N_z = 1$), $\hbar\omega_M = 4\pi\gamma M_s$ (for YIG, $\hbar\omega_M \approx 0.236$ K at room temperature), and $\hbar\omega_H = \gamma H_i$. The form of the dimensionless function \mathcal{F}_q depends on the direction of \mathbf{q} . In the important case when \mathbf{q} is parallel to H_o , i.e., along \hat{x} , one has

$$\mathcal{F}_q \rightarrow (1 - e^{-q_x d}) / (q_x d), \quad (3)$$

In the experiment [1] magnons are argued to condense at the minima $q_x = Q$; when $d = 5 \mu m$, and $H_o = 700$ Gauss, one has $|Q| \approx 5.5 \times 10^4 \text{ cm}^{-1}$.

We now set up a theoretical description of the BEC, including all 4-magnon scattering processes (3-magnon scattering is excluded by the kinematics), using a generalized Bogoliubov quasi-average technique [8] to incorporate the BEC. Defining magnon operators $b_{\mathbf{q}}, b_{\mathbf{q}}^\dagger$, a magnon BEC at $q = Q$, with N_o condensed magnons, has quasi-averages

$$\langle b_{\pm Q} \rangle = \langle b_{\pm Q}^\dagger \rangle = \sqrt{N_o/2}, \quad (4)$$

corresponding to a condensate wave-function $\Psi_Q(y) \propto \cos(Qy)$. More generally $\Psi_Q(y)$ is multiplied by a phase factor $e^{i\phi(\mathbf{r},t)/\hbar}$, which is crucial to the BEC dynamics, but not necessary for a stability analysis of the BEC.

We make a Holstein-Primakoff magnon expansion [9] up to 4th-order in magnon operators, including contributions from both (2 in - 2 out) and (3 in - 1 out) magnon scattering processes [10] (we ignore multiple-scattering contributions here, which are $\sim O(U_d/J_o) \sim 10^{-3}$ relative to the leading terms). Then, taking quasi-averages, we can write the Hamiltonian in the form

$$\mathcal{H} = \hbar \sum_{\mathbf{q}} \omega_{\mathbf{q}} (b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + \frac{1}{2}) + \hat{V}_{int}^p + \hat{V}_{int}^{-p} \quad (5)$$

where the interaction term \hat{V}_{int}^p takes the form

$$\begin{aligned} \hat{V}_{int}^p = & n_0 (\Gamma_0 + \frac{\Gamma_S}{4}) \sum_{p < Q} [b_{Q+p}^\dagger b_{Q+p} + b_{-Q-p}^\dagger b_{-Q-p}] \\ & + \frac{\Gamma_S n_0}{4} \sum_{p < Q} [b_{Q+p}^\dagger b_{-Q+p} + b_{-Q+p}^\dagger b_{Q+p} + \\ & \quad + b_{Q+p} b_{-Q-p} + b_{Q+p}^\dagger b_{-Q-p}^\dagger] \\ & + \frac{\Gamma_0 n_0}{2} \sum_{p < Q} [b_{Q+p} b_{Q-p} + b_{-Q-p} b_{-Q+p} + \\ & \quad + b_{Q+p}^\dagger b_{Q-p}^\dagger + b_{-Q-p}^\dagger b_{-Q+p}^\dagger]. \end{aligned} \quad (6)$$

Here Γ_0 and Γ_S are the four-magnon scattering amplitudes between states $(\mathbf{Q}, \mathbf{Q}) \rightarrow (\mathbf{Q}, \mathbf{Q})$ and $(\mathbf{Q}, -\mathbf{Q}) \rightarrow (\mathbf{Q}, -\mathbf{Q})$ respectively. For the sample geometry in Fig.1, with $\mathbf{H}_o \parallel \hat{x}$, these scattering amplitudes are found to be

$$\Gamma_0 = -\frac{\hbar\omega_M}{8S} [(\alpha_1 - \alpha_3)\mathcal{F}_Q - 2\alpha_2(1 - \mathcal{F}_Q)] - \frac{JQ^2}{4S} [\alpha_1 - 4\alpha_2]; \quad (7)$$

$$\Gamma_S = \frac{\hbar\omega_M}{2S} [(\alpha_1 - \alpha_2)(1 - \mathcal{F}_Q) - (\alpha_1 - \alpha_3)\mathcal{F}_Q] + \frac{JQ^2}{S} [\alpha_1 - 2\alpha_2], \quad (8)$$

with $\alpha_1 = u_Q^4 + 4u_Q^2 v_Q^2 + v_Q^4$, $\alpha_2 = 2u_Q^2 v_Q^2$ and $\alpha_3 = 3u_Q v_Q (u_Q^2 + v_Q^2)$, where $\{u_q, v_q\} = [(A_q \pm \hbar\omega_q)/2\hbar\omega_q]^{1/2}$ and A_q and B_q are given by $A_q = \gamma H_i + Jq^2 + 2\pi\gamma M_s \mathcal{F}_q$ and $B_q = -\pi\gamma M_s \mathcal{F}_q$ respectively. Higher-order multiple-scattering contributions to Γ_0, Γ_S are $\sim O(U_d/J_o)$ relative to the leading terms given here.

We plot the combinations $2\Gamma_0 \pm \Gamma_S$ in Figures 2 and 3. Both these amplitudes are sensitive to the external field and the film thickness. The first amplitude, $2\Gamma_0 + \Gamma_S$, becomes negative in the entire region of fields at $d < d_c \approx 2(J/\hbar\omega_M)^{1/2} \approx 0.032 \mu m$. The second amplitude is positive at $d < d_c$.

Near the energy minimum (when $|p| \ll |Q|$), one has $\omega_{Q+p} \approx \omega_{Q-p}$, and the Bogoliubov transformation is

straightforward because \mathcal{H} is symmetric when $p \leftrightarrow -p$ and $Q \leftrightarrow -Q$. The spectrum thus has 4 branches, with excited quasiparticle energies

$$\epsilon_{p\eta} = \sqrt{\Omega_Q(p)[\Omega_Q(p) + n_o(2\Gamma_0 + \eta\Gamma_S)]} \quad (9)$$

where $\eta = \pm 1$, and $\Omega_Q(p) = \hbar(\omega_{Q+p} - \omega_Q)$.

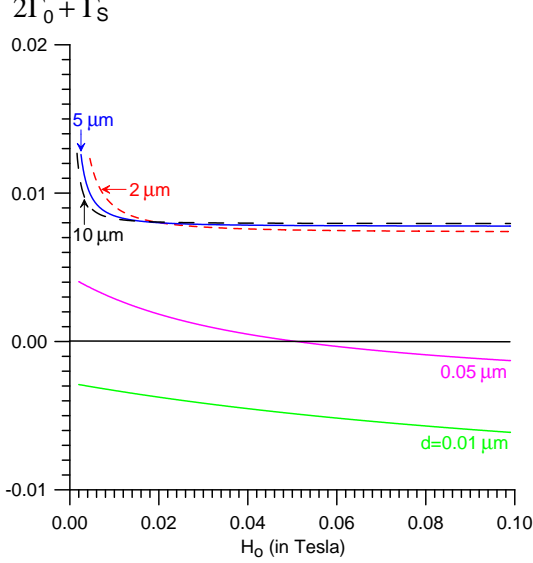


FIG. 2: (color online). The effective amplitude $2\Gamma_0 + \Gamma_S$ (in Kelvins) as a function of field H_o at different values of the film thickness d . Dashed lines: $d = 2$ and $10 \mu m$. Solid lines: $d = 0.01$, 0.05 and $5 \mu m$.

The inset in Fig.3 shows the "phase diagram" of the quasi-two dimensional YIG for different values of d and H_o^x . One immediately sees a paradox: the amplitudes are never both positive, so bulk BEC should not exist - yet BEC has been observed [1] in samples with an in-plane field $H_o^x \sim 0.07 T$, in which $d \sim 5 \mu m$.

The paradox is resolved by noting that in a finite geometry, the energy gap from the condensate to excited modes can be larger than the scattering amplitude, leading to a potential barrier to decay. For weakly-interacting Bose gases this yields [11] an upper critical number n_o^{cr} of condensate particles, above which the barrier disappears; one has $\alpha_o n_o^{cr}/l_o = k$, where α_o is the s-wave scattering length, and l_o the characteristic size of the BEC wave function. The constant $k \sim O(1)$, and depends on the sample geometry.

In the present case we can write the critical density n_o^{cr} as $|2\Gamma_0 - \Gamma_S| n_o^{cr} \sim \epsilon_p^{min}$, where ϵ_p^{min} is the minimum quasiparticle energy in the presence of the BEC; below this critical density the BEC is metastable to tunneling or thermal activation. If the BEC were to spread through the entire slab, then $n_o^{cr}|2\Gamma_0 - \Gamma_S| = \Omega_Q(\pi/L)$, where the length L depends on the direction of the field relative to the slab axes. Taking this result literally for the experiment [1], with a slab measuring $20 \times 2 mm^2$ in the plane, one finds $10^{-8} < n_o^{cr} < 10^{-6}$ (for fields along the

long and short sides of the slab respectively). However this result is certainly too low, since it assumes a perfectly uniform BEC - in reality disorder and edge effects will smear the magnon spectrum and restrict the size of the BEC, and a more realistic estimate for L is then $L_{eff} = (L_x L_y L_z)^{1/3} \sim 0.6 mm$. This yields $n_o^{cr} \sim 10^{-5}$ for the experiment. Thus we conclude that for in-plane fields, it will be impossible to raise n_o above this value; this could be checked experimentally (eg., by increasing the pumping rate).

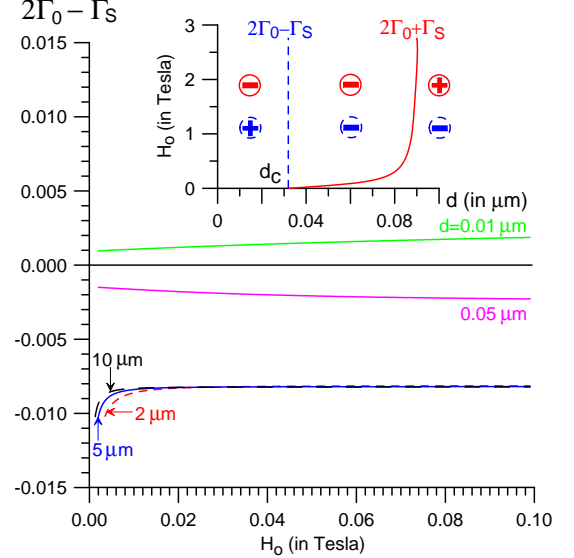


FIG. 3: (color online). The effective amplitude $2\Gamma_0 - \Gamma_S$ (in Kelvins) as a function of external field H_o at different values of the film thickness d . Dashed lines: $d = 2$ and $10 \mu m$. Solid line: $d = 0.01$, 0.05 and $5 \mu m$. The inset shows the "phase diagram" of YIG in the (H_o, d) plane. The amplitude $2\Gamma_0 + \Gamma_S$ is positive everywhere to the right of the solid line (the region \oplus). The amplitude $2\Gamma_0 - \Gamma_S$ is negative everywhere to the right of the dashed line (the region \ominus).

(ii) Perpendicular field: Now the results are very different. Consider first an infinite thin slab, which is simple to analyze. The competition between the external field $\mathbf{H}_o = \hat{z}H_o$ and the demagnetization field (which favours in-plane magnetisation) gradually pulls the spins out of the plane; below a critical field $H_c = \hbar\omega_M/\gamma \approx 1760 G$, the Free Energy is degenerate with respect to rotation around \hat{z} and so the magnons are gapless, but at H_c they align with \mathbf{H}_o and a gap $\hbar\omega_o \equiv \hbar\omega_{q=0} = \gamma H_o - \hbar\omega_M$ opens up. The minimum in the magnon spectrum is always at $q = 0$ (Fig.4).

The inter-magnon scattering amplitude Γ is now always positive; neglecting a very small exchange contribution one finds

$$\begin{aligned} \Gamma(\mathbf{q}) &= (\hbar\omega_M/4S)[1 - (1 - \mathcal{F}_q)/2] \\ &\rightarrow (\hbar\omega_M/4S)[1 - q_{||}d/4 + O(q_{||}^2)] \end{aligned} \quad (10)$$

where $q_{||}$ is the momentum in the xy plane, and \mathcal{F}_q takes the form (3) but with $q_x \rightarrow q_{||}$. This radically changes

the situation - now a BEC is stable with no restriction on the condensate density. There are however restrictions on H_o ; when $2200\text{ G} < H_o^z < 3500\text{ G}$ the system has a “kinetic instability” [12], in which the pumping of the magnons at one frequency destabilizes the magnon distribution, along with strong microwave emission.

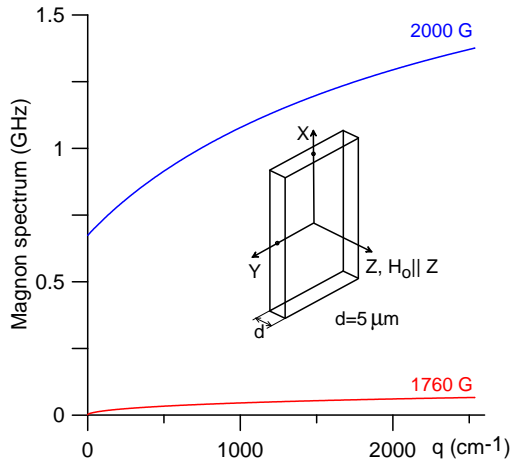


FIG. 4: (color online). The magnon spectrum for an infinite slab of YIG, assuming free surface spin boundary conditions, with magnetisation polarised perpendicular to the plane (see inset). The spectra are shown for $H_o^z = H_c \approx 1760\text{ G}$ where the spectrum is gapless, and for $H_o^z = 2000\text{ G}$.

In a real finite sample things are much more complicated. Even without surface anisotropy the spins near the slab edge are put out of alignment with the bulk spins by edge demagnetisation fields; and surface anisotropy does the same to spins on the slab faces. However in the central region of the sample, at distances further from the surface than the exchange length, the spectrum returns to the infinite plane form. For fields well above H_c , eg., for $H_o^z \sim 2000\text{ G}$, all of the spins will be aligned along \hat{z} , and (10) will then be valid everywhere.

In this case we have the striking result that a BEC of pumped magnons should be possible with densities much higher than present. To give an upper bound is complicated since the problem then becomes essentially non-perturbative (similar to, eg., liquid ^4He), beyond the range of higher-order magnon expansions. However there appears to be no obstacle in principle to raising $n_o/n \sim O(1)$. At present the highest achievable density is probably limited by experimental pumping strengths rather than any fundamental restrictions. Such a high-density BEC existing at room temperature would be extremely interesting, and certainly possess unusual magnetic properties.

Remarks: The 2 cases studied above are actually limiting cases of a more general situation in which one can manipulate the inter-magnon interactions by varying the field direction and strength, and vary the upper critical density for BEC formation by changing the sample

geometry. Thus the analysis can be easily generalized to long ‘magnetic wires’ or whiskers, and we also expect that BEC will be stabilized there when the external field is perpendicular to the sample axis, but unstable or metastable when the field is parallel to the sample axis. Further details of the various possible cases will be published elsewhere.

Acknowledgements: We acknowledge support by the Pacific Institute of Theoretical Physics, the National Science and Engineering Council of Canada, the Canadian Institute for Advanced Research, the Louisiana Board of Regents (Contract No. LEQSF (2005-08)-RD-A-29), the Tulane University Research and Enhancement Fund, and the US Air Force Office of Scientific Research (Award no. FA 9550-06-1-0110). We would also like to thank B. Heinrich and D. Uskov for very useful conversations.

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